Projecting the population growth of African Rhinos granted their protection in Africa continues using logistic growth curves

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The African Black Rhino

- A subspecies of Rhino Native to Eastern Africa, having a specifically dense population in **Kenya** and **Tanzania**
- Current population stands at 6 421(STR organization)
- One of two Rhino species found in Africa, making its conservation incredibly significant
    - Both Rhino species have seen an increase in population sizes thanks to successful conservation efforts(WWF)

Why the African Black Rhino?

- In **1970,** Black Rhino populations soared, at a natural population rate of 37800 Rhinos
- In **1995,** the Rhino population reached an all time low of 2 410 individuals - a population decrease of 96% - due to the **ivory trade** and the value of their tusks
- In **1995**, their protection began: ivory hunters were heavily penalized and countries came together to employ protectors throughout the parks in order to ensure that the species would not go extinct
- After the beginning of their protection, the Rhinos have been steadily increasing in population size - as of late, there are 6 421 Black Rhinos throughout the park

Why the specific subspecies:

- Some sub-species have not felt the same protection as the Black Rhino, leading to their eventual extinction. Focusing on the population growth of one specific subspecies is more realistically quantized

African Black Rhino conservation has been, to a large extent, succesful since its implementation in 1995. I have therefore chosen this animal to be the subject of my investigation, to quantize how these continuous conservation efforts can have proven succesful, and how Black Rhino populations will continue to change in later years, granted their protection continues

Year	**t**	P_R
1995	0	2354
1999	4	2700
2001	6	3100
2005	10	3726
2007	12	4227
2009	14	4879
2010	15	4880
2012	17	4845
2013	18	5250
2015	20	5214
2017	22	5496
2021	26	6195
2022	27	6195
2023	28	6487

Figure 1: raw data

Our World in Data. "Black Rhino Population." *Estimated Number of Black Rhinos*, ourworldindata.org/grapher/black-rhinos?tab=table&time=earliest..2012. Accessed 12 Dec. 2024.

The General Equation

$$P(t) = \frac{L}{1+ae^{-kt}}$$

Where

P = population

t = years

L, a, k are constants to be solved

Further solving and mathematical theory behind this can be found in **supplementary material**

Assumptions necessary for the given calculation

- *Assuming the given data is accurate:*
    - Rhinos are wild animals, and completely accurate population estimates can be difficult to assess. This project assumes that the given data is correct
- *Assuming the Carrying Capacity of Black Rhinos is 37 800:*
    - True carrying capacity is difficult to assess, as there is no correct measure of it. The largest recorded Black Rhino population, before their poaching and population decline, was recorded to be 65 000, and therefore this is a fair estimate to carrying capacity. It is also important to note that changing
- *Assuming there are no external barriers to Rhino populations:*
    - This is a key limitation of the mathematical model: the unpredictability of populations. Poaching and the ivory trade is just one factor effecting the population size of Rhinos, and today the threat of habititat damage due to global warming, enviornemental degradation, disease, and a variety of unpredictable variables may threaten population sizes. This mathematical model assumes that these variables are negligible, which **does not** reflect the reality of population fluctuations

Solving for constants *a* and *L*

Maximum growth capacity, ***L*,** or carrying capacity is the maximum population which a habitat can sustain, and is a naturally occurring phenomena. For the sake of the investigation, it is assumed that the maximum growth capacity of the Black Rhino is 37 800. This is a fair assumption because that was their maximum recorded population before they faced natural threats and became subject to the ivory trade and poaching. However, it must be noted that 37800 is only an assumption and the true maximum carrying capacity may vary far from this assumption.

Therefore,

L = 37800

a is a constant specific to the formulae, and therefore can be found simply by plotting two data points from the observed data. To find *a*, I plotted the first given data plot, the year 1995 at which:

$t = 0$

$P(t) = 2354$

From these data points I was able to find:

$$\frac{37800}{2354} - 1 = a$$

$$a = 15.057774$$

Further working can be found in the essay and supplementary material.

Solving for k

Year	t	P_R	k - value[7]
1995	0	2354	none
1999	4	2700	0.03673626131
2001	6	3100	0.04942613787
2005	10	3726	0.04986950447
2007	12	4227	0.0533053
2009	14	4879	0.05733742368
2010	15	4880	0.05353061645
2012	17	4845	0.04674697962
2013	18	5250	0.04929695059
2015	20	5214	0.0439679485
2017	22	5496	0.04276017709
2021	26	6195	0.04162775315
2022	27	6195	0.04008598451

***Figure 2:* finding *k* values**

The constant, *k*, in the general equation represents the **growth constant** of population growth. Similarly to finding the constant *a*, the *k* constant must be worked through backwards-working the general equation. As a result of the external factors effecting Rhino population growth, the k-value varies slightly for each year: as seen through the k - value row of the diagram to the left. To ensure maximum accuracy, I took the mean of each calculated *k-value*, to find the most general value. Although the difference between *k-values* is relatively small to the final values of the equation, it is crucial to be as accurate as possible. **The full working for *k* can be found within the project.** The man of the found *k-values* is found to be:

$$k = 0.04705758644$$

Inserting the found constants gives us:

$$P(t) = \frac{L}{1+ae^{-kt}}$$

$$P(t) = \frac{37800}{1+14,93033135e^{-0.04705758644t}}$$

Year	t	P_R	$P(t)$ [9] Rounded to be a whole number	Error $P_R - P(t)$
1995	0	2354	2373	-19
1999	4	2700	2828	-128
2001	6	3100	3084	16
2005	10	3726	3661	65
2007	12	4227	3983	244
2009	14	4879	4332	547
2010	15	4880	4516	364
2012	17	4845	4904	-59
2013	18	5250	5108	142
2015	20	5214	5108	106
2017	22	5496	5998	-502
2021	26	6195	7010	-815
2022	27	6195	7282	-1087
2023	28	6487	7563	-1076

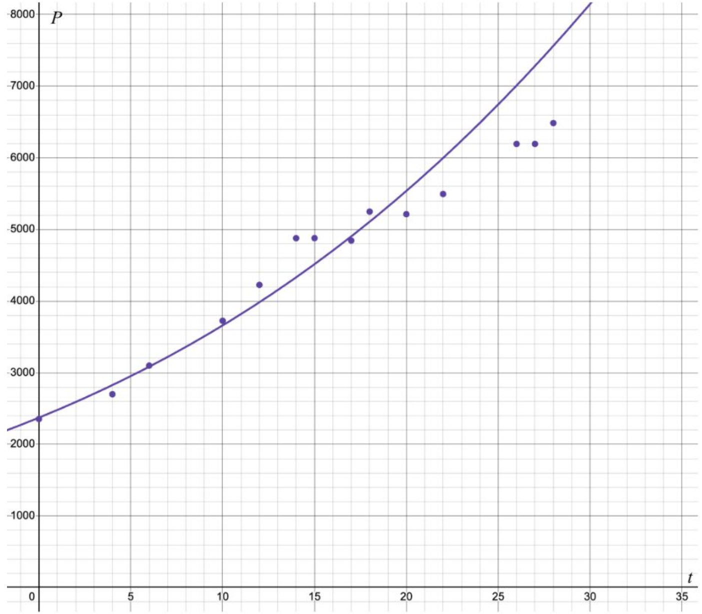
Figure 3.1: Accuracy of the Function

P(t) vs Raw Data

$$MAE = \frac{1}{n}\sum_{i=0}^{n}\left|P_{Ri} - P(t)_i\right|$$ [10]

$$= \frac{19+128+16+65+244+547+364+59+142+106+502+815+1087+1076}{14}$$

$$= 369.2857143 \approx 369.3$$



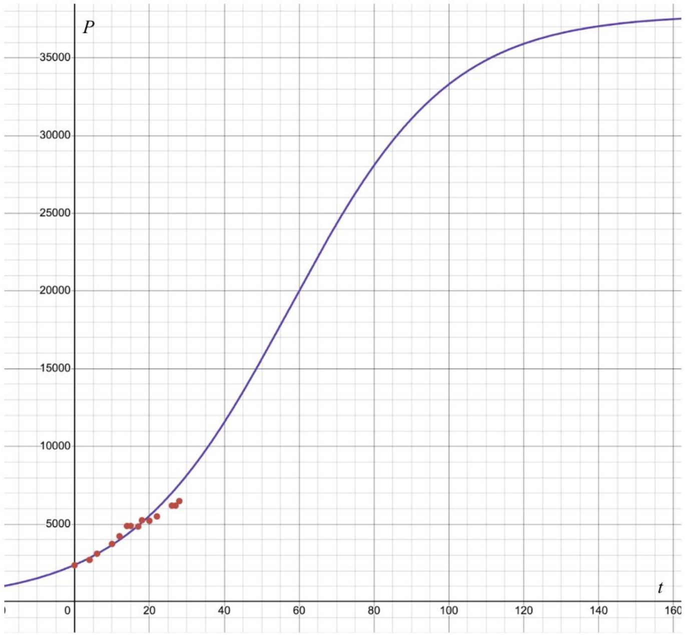
The significance of this data:

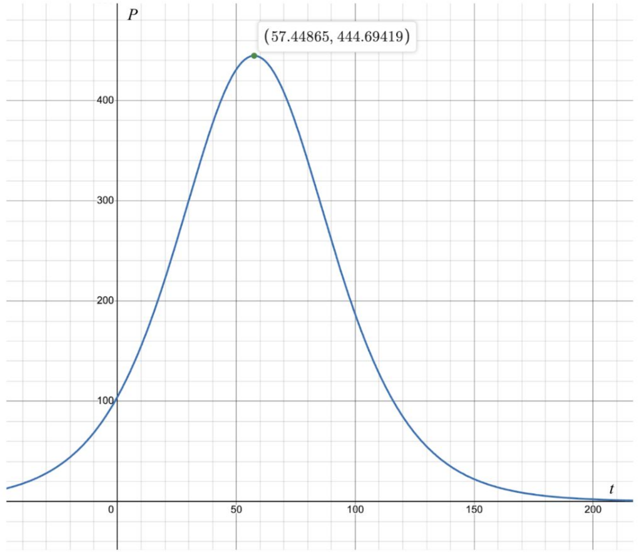
- It is important to note that each given value for *P(t)* is rounded to the nearest whole number, as population size represents discrete data, meaning it can only be in integers
- The data has a mean average error of ±369.3 rhinos, which is a relatively significant error compared to the population sizes themselves
    - This error is due to the limited ability to mathematically account for biological factors effecting population sizes. Rhinos, being wild animals, are subject to significant and unpredictable events effecting their population rate, which is a major limitation of the model as explained under the "assumptions" slide
    - **If** population sizes were to be experimentally regulated however, in a controlled system, this model would provide relatively accurate predictions of future population growth and therefore this model holds reliable, its limitations taken into account

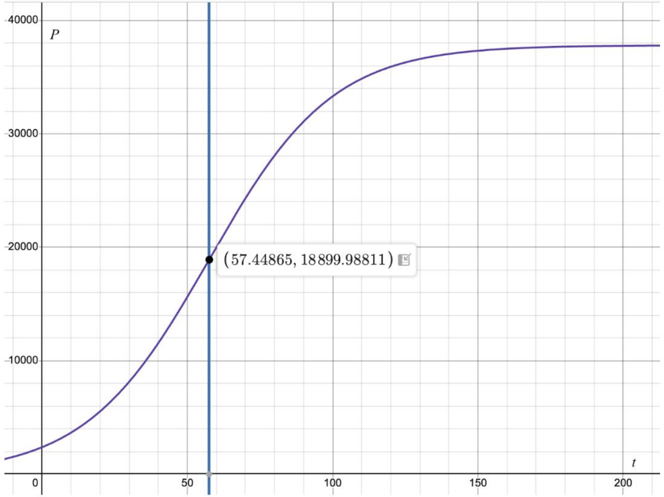
Year	t	$P(t)$
2025	30	8149
2030	35	9752
2035	40	11549
2040	45	13517
2045	50	15621
2050	55	17812
2055	60	20033
2060	65	22223
2065	70	24325
2070	75	26290
2075	80	28082
2080	85	28082
2085	90	31082
2090	95	32285
2095	100	33304
2100	105	34155
2105	110	34860
2110	115	35438
2115	120	35908
2120	125	36289
2125	130	36596
2130	135	36842
2135	140	37039
2140	145	37196
2145	150	37321
2150	155	37420
2155	160	37499
2160	165	37562
2165	170	37612
2170	175	37650
2175	180	37682
2180	185	37706
2185	190	37726
2190	195	37742
2195	200	37754
2200	205	37764
2205	210	37764
2210	215	37771
2215	220	37777
2220	225	37782
2225	230	37789
2230	235	37791
2235	240	37793
2240	245	37794
2245	250	37796
2250	255	37797
2255	260	37797
2260	265	37798
2265	270	37798
2270	275	37799
2275	280	37799
2280	285	37799
2285	290	37799
2290	295	37799
2295	300	37800
2300	305	37800
2305	310	37800
2310	315	37800
2315	320	37800

The results

P(t) function graphed using DESMOS calculator







Graphical Analysis of the Data: the rate of change

- Graphing the curves derivative, **P(t)’** shows us how the rate of population growth changes over time
  - This is a major benefit of choosing the population growth model to map growth as opposed to an exponential model which does not take into account the changing population growth over time
- As seen from the data, there occurs a point of inflection at $t \approx 57$, signifying that there is a maximum change in the population growth, and from this point onwards, $t > 57$, population rates begin decreasing as populations are hit with limitations from the carrying capacity
- This resembles natural phenomenon: fast growth can only be sustained until the resources from habitats become relatively depleted

The beauty of this population model & moving forward

Data on animal populations is crucial to understanding how species can be protected. This mathematical model provides a basis for calculating the worth of conservation of critically endangered species, as it provides clear and accurate estimates of projected population growth which acts as a reminder of the importance of preserving our planets biodiversity. With the right data, this model could be easily adapted to any species of animal to estimate future population values. This model could also be used to estimate extinction dates in the case of an animal having negative population growth, which could help foster worldwide responsibility and create a sense of urgency to protect the species in critical endangerment. This mathematical model is vastly applicable and reliable, and therefore should be used as a basis for arguing in favor of animal conservation and protection, despite its limitations.